Comment on “Statistical Mechanics of Non-Abelian Chern-Simons Particles”

by

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The second virial coefficient $B_2(T)$ for particles which interact with each other through a Chern-Simons type coupling has been of considerable interest in recent years. In particular its form has been derived for both the spin zero [1] and spin-1/2 cases [2] as a function of the flux parameter α. More recently results have been obtained for the case of a gas of spinless non-Abelian Chern-Simons particles. [3] It is the purpose of this note to point out that the form of the virial coefficient obtained in that work for the SU(2) case is quantitatively incorrect and also that there exists a periodicity in the flux parameter just as in the Abelian theory. No such periodicity was noted in ref. 3.

To demonstrate the result one can begin with Eq. (25) of ref. 3 which reads

$$H'_j = -\frac{1}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left(\frac{\partial}{\partial\theta} + i\omega_j\right)^2\right]$$

where ω_j can (by trivial scaling of the interaction parameter) be taken to be

$$\omega_j = 2\alpha[j(j+1) - 2\ell(\ell+1)] \tag{1}$$

where $\ell = 0, \frac{1}{2}, 1, \ldots$ and j is any allowed angular momentum (i.e., $j = 0, 1, 2, \ldots 2\ell$). One infers the second virial coefficient from the expression

$$B_2(\alpha, T) = \frac{1}{(2\ell+1)^2}\sum_{j=0}^{2\ell}(2j+1)\left[\frac{1+(-1)^{j+2\ell}}{2}B_2^B(\omega_j, T) + \frac{1-(-1)^{j+2\ell}}{2}B_2^F(\omega_j, T)\right] \tag{2}$$

where superscripts B and F have been used to denote the virial coefficients for the Abelian bosonic and fermionic cases, respectively. It should be noted that the factors of $(-1)^{2\ell}$ have been improperly omitted in ref. 3. These factors are necessary to ensure that the symmetric [antisymmetric] isospin states contribute only to $B_2^B(\alpha, T)[B_2^F(\alpha, T)]$.

One now makes use of the well known results for spin zero particles

$$B_2^{B(F)}(\alpha, T) = \frac{1}{4}\lambda_T^2 \begin{cases} -1 + 4\delta - 2\delta^2 & \text{N even (odd)} \\ 1 - 2\delta^2 & \text{N odd (even)} \end{cases}$$

where λ_T is the thermal wavelength and $\alpha = N + \delta$ with N an integer such that $0 \leq \delta < 1$. This leads upon insertion into (2) the equation

$$\frac{4}{\lambda_T^2} B_2(\alpha, T) = \mp \frac{1}{2\ell+1} + \frac{2}{(2\ell+1)^2} \sum_0^{2\ell} (2j+1)\delta_j [1 \pm (-1)^{j+2\ell} - \delta_j] \tag{3}$$

where δ_j is given by

$$\omega_j = N_j + \delta_j$$

with N_j an integer and ω_j given by Eq. (1). The upper and lower signs in (3) refer to the cases of even and odd N_j respectively. [4] It is important to note that the definition of ω_j implies for all j that ω_j is an integer multiple of α.

One immediate point of contrast between (3) and the results of ref. 3 is that for $\alpha \to 0$ the latter predicts that the virial coefficient becomes the free bosonic result (see Eq. (30) of ref. 3). This could only be correct if the configuration space wave functions for all states were symmetric. In fact those states with angular momentum $j-1$, $j-3$, etc. necessarily have antisymmetric wave functions and thus give a contribution to $B_2(\alpha = 0, T)$ which is equal and opposite to that of the bosonic case. In fact a simple calculation for the free particle case easily confirms the $\alpha = 0$ prediction of Eq. (3).

A final observation has to do with the periodicity issue. Since, as observed earlier, ω_j is an integer multiple of α, it follows that the contribution of each j to $B_2(\alpha, T)$ is periodic, with each recurrence occurring with (at most) a change of two units in α. For specific ℓ values the period can, of course, be seen to be reduced to a shorter interval. Clearly the total virial coefficient also satisfies this periodicity condition, thereby showing that there is little qualitative difference between the Abelian and non-Abelian cases.

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References

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2. T. Blum, C. R. Hagen, and S. Ramaswamy, Phys. Rev. Lett. **64**, 709 (1990).

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4. It is worth noting that in ref. 3 no differentiation is made between even and odd N_j. This is not permissible even in the Abelian case.